Lesson 5-2: Bisectors in Triangles

What's up with all the triangle stuff?

You may be thinking "Come on, let's get on with it! Enough with the triangles!" Well I can understand if you are feeling that way. But bear with us, because as I've said before, we will be using what we know about triangles to help us with more complex figures later.

Segment bisector

You know this: a *segment bisector* is a line that passes through the midpoint of that segment.

You also know this: a *perpendicular bisector* of a line segment is a line that divides the line segment into two congruent parts (bisects it) and that is also perpendicular to the line segment.

Let's play with this a bit:

1. On a clean sheet of paper, draw a segment, label it $\overline{PQ}$.
2. Fold your paper over so that endpoints *P* and *Q* coincide (land exactly on top of each other). Crease your paper along the fold.
3. Unfold your paper. Draw a line in the crease. Does the line appear to be the perpendicular bisector of $\overline{PQ}$? Check with your ruler and protractor to verify that the line in the crease is indeed the perpendicular bisector of $\overline{PQ}$.

   How would you describe the relationship of the points on the perpendicular bisector with the endpoints of the bisected segment?
4. Place three points on your perpendicular bisector. Label them *A, B, & C*. With your compass, compare the distances *PA* and *QA*. Compare the distances *PB* and *QB*. Compare the distances *PC* and *QC*.

   What do you notice about the two distances from each point on the perpendicular bisector to the endpoints of the segment? State a conjecture.

Perpendicular bisector conjecture

I suspect your conjecture is something like the following:

> If a point is on the perpendicular bisector of a segment, then it is the same distance from each of the endpoints of the segment.

The only thing I'd change is to say "equidistant" rather than "the same distance" … sounds more "mathematical" doesn't it? ☺

So, we have a conjecture; what do we do with it? That's right! We prove it and come up with a new theorem!

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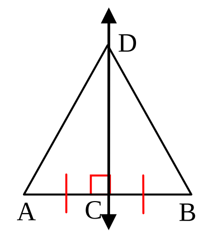
Proof

Given: $\overleftrightarrow{CD} \perp \overline{AB}, \overleftrightarrow{CD}$ bisects $\overline{AB}$
Prove: $DA = DB$



Proof:	$\overleftrightarrow{CD} \perp \overline{AB}$	(Given)
	$\overline{AC} \cong \overline{BC}$	(defn. bisector)
	$\angle DCA \,\&\, \angle DCB$ are rt $\angle$'s	(defn. perpendicular)
	$\angle DCA \cong \angle DCB$	(all rt. $\angle$'s are $\cong$)
	$\overline{DC} \cong \overline{DC}$	(Reflexive POC)
	$\Delta ADC \cong \Delta BDC$	(SAS)
	$\overline{AD} \cong \overline{BD}$	(CPCTC)
	$DA = DB$	(defn. congruence)
	Q.E.D.	

Theorem 5-2 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Now, can you form the converse of this statement?

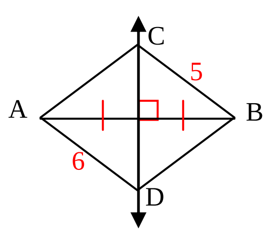
Theorem 5-3 Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is the perpendicular bisector of the segment.

The proof for this theorem is similar to that of Theorem 5.2. You can prove the two right triangles congruent by HL, then use CPCTC to prove the base legs ($\overline{AD} \,\&\, \overline{BD}$) are congruent.

Example – Pg 250 Check Understanding 1

Use the information given in the diagram. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$. Find *CA & DB*. Explain your reasoning.



CA = 5; DB = 6;

$\overleftrightarrow{CD}$ is the $\perp$ bisector of $\overline{AB}$, therefore $CA = CB$ and $DA = DB$.

Lesson 5-2: Bisectors in Triangles

Definition – Distance from a point to a line

A great way to think about the distance of a point to a line is to visualize the length of the *shortest* segment from the point to the line. If you experiment with it you will find that the shortest segment is perpendicular to the line. Thus:

> The distance from a point to a line is the length of the perpendicular segment from the point to the line.

Which leads us to…

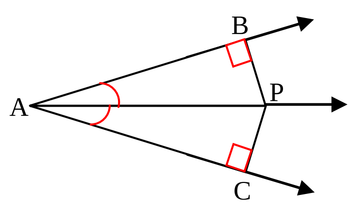
Theorem 5-4 Angle Bisector Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

Proof

Given: $\overline{PB} \perp \overrightarrow{AB}, \overline{PC} \perp \overrightarrow{AC}, \overrightarrow{AP}$ bisects $\angle BAC$

Prove: $PB = PC$



Proof: $\angle PBA \cong \angle PCA$	(all rt. $\angle$'s are $\cong$)
$\angle BAP \cong \angle CAP$	(defn. angle bisector)
$\overline{AP} \cong \overline{AP}$	(reflexive POC)
$\Delta BAP \cong \Delta CAP$	(AAS)
$\overline{PB} \cong \overline{PC}$	(CPCTC)
$PB = PC$	(defn. of segment congruence)
Q.E.D.	

Theorem 5-5 Converse of the Angle Bisector Theorem

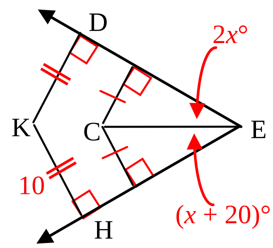
If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the bisector of the angle.

Note the important qualifier: "in the interior of an angle."

Again, the proof of 5-5 is similar to that of 5-4. We prove the triangles congruent by HL and then use CPCTC to show the angles congruent and therefore bisected.

Lesson 5-2: Bisectors in Triangles

Example – Pg. 251 Check Understanding 2



a) According to the diagram, how far is K from $\overrightarrow{EH}$? 10
From $\overrightarrow{ED}$? 10

b) What can you conclude about $\overrightarrow{EK}$?
It is the angle bisector of $\angle DEH$.

c) Find the value of x.
$2x = x + 20$
$x = 20$

d) Find $m\angle DEH$.
$2*x + (x + 20) = 2*20 + (20 + 20) = 80$

Assign homework

p. 251 #1-25, 35-39 odd, 44, 46, 50-52